## THE MANGA GUIDE" TO

## LINEAR <br> 

SHIN TAKAHASHI
IROHA INOUE TREND-PRO CO., LTD.

## INDEX

SPECIAL CHARACTERS AND NUMBERS

3-D projections of linear transformations, 185
$\theta$ (theta), 180

## A

addition
with matrices, 70
with vectors, 125
axis, expressing with vectors, 127

## B

basis, 140-148, 156-158
binomial coefficients, 60

## $c$

co-domain, 39, 45
cofactor matrices, 110
cofactors, calculating inverse matrices using, 88, 108-111
column vectors, 126
combinations, 55-60
complex numbers, 25
computer graphics
systems, linear
transformations
used by, 184
conventional linear transformations, 184
coordinates, 161-162
Cramer's rule, 111-112

## D

dependence, linear, 135, 138-139, 143
determinants calculating, 96-105, 111-112
overview, 95
diagonalization, multiplicity and, 224-229
diagonalizing matrices, 221, 225
diagonal matrices, 80-81
dimensions, 149-162
dimension theorem
for linear
transformations, 189-192
domain, 39, 44-45

## E

eigenbasis, 229
eigenvalues
calculating, 216-218
finding $p$ th power of $n \times n$ matrix, 219-221, 224-229
overview, 210-215
relation of linear algebra to, 24
eigenvectors
calculating, 216-218
finding $p$ th power of $n \times n$ matrix, 219-221, 224-229
overview, 210-215
relation of linear
algebra to, 24
elementary matrices, 196
elements
in matrices, 67
in sets, 30,32
equations, writing as
matrices, 69
equivalence, 29

## F

functions
defined, 39
domain and range, 44-45
and images, 40-43
inverse, 48-49
linear transformations, 50-61
onto and one-to-one, 46-47
overview, 35-39
$f(x), 40-43$

## G

Gaussian elimination, 88-89, 91, 108
geometric interpretation, of vectors, $127-130$
graphs, of vectors, 144

## I

$i$ (imaginary unit), 25-26
identity matrices, 82-84, 92
images
and functions, 40-44
overview, 174, 189-192
imaginary numbers, 25
imaginary unit (i), 25-26
implication, 27-28
independence, linear, 132-139, 143, 146-147
integers, 25
inverse functions, 48-49
inverse matrices
calculating using
Gaussian elimination, 88-94
calculating using cofactors, 108-111
overview, 86-87
invertible matrices, 94
irrational numbers, 25

## $K$

kernel, 189-192
L
linear algebra, overview, 9-20
linear dependence, 135, 138-139, 143
linear independence, 132-139, 143, 146-147
linear map, 167
linear operation, 167
linear spans, 154-155
linear systems, solving with Cramer's rule, 111-112
linear transformations
3-D projections of, 185
applications of, 173-177
dimension theorem for, 189-192
functions and, 50-61
overview, 166-173
rank, 193-203
relation of linear algebra to, 24
relationship with matrices, 168, 203
rotation, 180-181
scaling, 179
translation, 182-184
lower triangular
matrices, 79

## M

main diagonal
diagonal matrices
and, 80
identity matrices and, 82
overview, 67
symmetric matrices and, 79
triangular matrices and, 79
matrices
calculations with, 70-76
determinants, 95-105, 111-112
diagonal, 80-81
diagonalizable, 225-227
eigenvalues and eigenvectors, 215
identity, 82-84
inverse
calculating using
Gaussian
elimination, 88-94
calculating using cofactors, 108-111
overview, 86-87
lower triangular, 79
multiplication with, 72-76, 125
overview, 62-69
rank of, 196-203
relation of linear algebra to, 24
relationship with linear transformations, 203
symmetric, 79
transpose, 78
upper triangular, 79
writing systems of equations as, 69
zero, 77
multiplicity, and diagonalization, 224-229
multiplication with diagonal matrices, 80-81
with identity matrices, 82-83
with matrices, 72-76
with vectors, 125

## N

natural order, 103
non-diagonalizable matrices, 227-229
number systems, 25-26

## 0

objects, in sets, 30
one-dimensional dependence, 135 , 138-139, 143
one-dimensional independence, 132-139, 143, 146-147
one-to-one functions, 46-47
onto functions, 46-47

## $P$

permutations, 55-60
perspective projection, 185
planes, 128
points, 127
polynomial roots, 224
propositions, 27
$R$
range, 44-45
rank
of matrices, calculating, 196-203
overview, 193-195
rational numbers, 25
real numbers, 25
$R^{n}, 126$
rotating linear transformations, 180-181, 184
row vectors, 126
rules
of determinants, 101
functions as, 39

## $S$

Sarrus' rule, 98
scalar multiplication
with matrices, 72
with vectors, 125
scaling linear transformations,
179, 184
set theory
sets, 30-31
set symbols, 32
subsets, 33-34
square matrices
multiplying, 75
overview, 67
straight lines, 127
subscripts, 66
subsets, 33-34
subspaces, 150-155
subtraction
with matrices, 71
with vectors, 125
symbols
for equivalence, 29
for functions, 39
$f(x), 40-43$
for imaginary units, 25-26
for inverse functions, 49
for propositions, 28
of sets, 32
for subsets, 33
for transpose matrices, 78
symmetric matrices, 79
systems of equations, writing as matrices, 69

## T

target set, 39
term indexes, 101
theta ( $\theta$ ), 180
3-D projections of linear transformations, 185
transformations, linear. See linear transformations
translating linear transformations, 182-184
transpose matrices, 78
triangular matrices, 79
U
upper triangular matrices, 79

## V

vectors
basis, 140-148
calculating, 125-126
dimensions of, 149-162
geometric interpretation of, 127-130
linear independence, 132-139
overview, 116-124
relation of linear algebra to, 24
vector space, 129

## Z

zero matrices, 77

